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Darboux integrating factors: Inverse problems ☆

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ABSTRACT

We discuss planar polynomial vector fields with prescribed Darboux integrating factors, in a nondegenerate affine geometric setting. We establish a reduction principle which transfers the problem to polynomial solutions of certain meromorphic linear systems, and show that the space of vector fields with a given integrating factor, modulo a subspace of explicitly known "standard" vector fields, has finite dimension. For several classes of examples we determine this space explicitly.

1. Introduction

Given a planar polynomial vector field, there is the classical problem (going back to Darboux and Poincaré) to determine an integrating factor of Darboux type, or to verify that no such factor exists. The papers and survey papers by Schlomiuk [15–17] give a good introduction to this field of research. A preliminary question is to determine the invariant algebraic curves of the vector field, or to ensure that no such curves exist. Several results, mostly for settings with certain additional conditions, are known; we mention Cerveau and Lins Neto [4], Carnicer [3], Camacho and Sad [2], Lins [11],

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