

Inverse Problems in Darboux' Theory of Integrability

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Abstract The Darboux theory of integrability for planar polynomial differential equations is a classical field, with connections to Lie symmetries, differential algebra and other areas of mathematics. In the present paper we introduce the concepts, problems and inverse problems, and we outline some recent results on inverse problems. We also prove a new result, viz. a general finiteness theorem for the case of prescribed integrating factors. A number of relevant examples and applications is included.

Keywords Planar polynomial vector field · Invariant curve · Elementary integrability · Integrating factor

1 Introduction and Preliminaries

Consider a planar complex polynomial vector field

$$X = P \frac{\partial}{\partial x} + Q \frac{\partial}{\partial y}, \quad (1)$$

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