



Invariant algebraic curves of large degree for quadratic system

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Abstract

In this paper we present for the first time examples of algebraic limit cycles and saddle loops of degree greater than 4 for planar quadratic systems. In particular, we give examples of algebraic limit cycles of degree 5 and 6, and algebraic saddle loops of degree 3 and 5 surrounding a strong focus. We also give an example of an invariant algebraic curve of degree 12 for which the quadratic system has no Darboux integrating factors or first integrals.

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1. Introduction

We shall study polynomial differential systems in $\mathbb{R}^2$ defined by

$$\dot{x} = p(x, y), \quad \dot{y} = q(x, y), \quad (1)$$

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