Nonlinear Dyn (2014) 75:561–566
DOI 10.1007/s11071-013-1085-3

ORIGINAL PAPER

Zero–Hopf bifurcation in a hyperchaotic Lorenz system

Lorena Cid-Montiel · Jaume Llibre · Cristina Stoica

Received: 5 July 2013 / Accepted: 14 September 2013 / Published online: 12 October 2013

Abstract We characterize the zero–Hopf bifurcation at the singular points of a parameter codimension four hyperchaotic Lorenz system. Using averaging theory, we find sufficient conditions so that at the bifurcation points two periodic solutions emerge and describe the stability of these orbits.

Keywords Hyperchaotic Lorenz system · Zero–Hopf bifurcation · Periodic orbits · Averaging theory

1 Introduction

In 1963, Edward Lorenz [19] introduced a system of ordinary differential equations in $\mathbb{R}^3$ soon to became famous for exhibiting chaotic solutions for certain parameter values and initial conditions. More precisely, the Lorenz system displays a set of chaotic solutions, which when plotted, looks as a butterfly or figure eight (the Lorenz attractor). The origins of this system lies in atmospheric modeling. However, the Lorenz equations also appear in the modeling of lasers; see [11] and for dynamos see [15].

Recently, a so-called *hyperchaotic Lorentz* system was introduced; see for instance [1, 6, 10, 13, 14, 25, 27–33] and the references therein. (MathSciNet presently lists 24 papers on *hyperchaotic Lorenz* systems.) We remark that not all these hyperchaotic Lorenz systems coincide, as they can vary in one or two terms. A precise definition of a hyperchaotic system comprises (1) an autonomous differential equations system with a phase space of dimension at least four, (2) a dissipative structure, and (3) at least two unstable directions, out of which at least one is due to a nonlinearity. Since it generates multiple positive Lyapunov exponents, the dynamics of hyperchaotic systems is hard to predict and control. Consequently, such systems are of use in secure communication, and thus received a great deal of attention mainly in engineering (circuit and communications systems; see, for instance [24], and references therein).

In this paper, we approach a hyperchaotic system from a dynamical systems point of view. More precisely, we investigate a *4-dimensional zero–Hopf equilibrium* (that is, an isolated equilibrium with a double zero eigenvalue and a pair of purely imaginary eigenvalues), and the birth of periodic solutions as parameters vary.

There are several works studying the unfolding of the 3-*dimensional zero–Hopf equilibria*. Recall

L. Cid-Montiel · C. Stoica (✉)
Department of Mathematics, Wilfrid Laurier University, Waterloo, N2L 3C5, Ontario, Canada
e-mail: cstoica@wlu.ca

L. Cid-Montiel
e-mail: cidx7550@mylaurier.ca

J. Llibre
Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Catalonia, Spain
e-mail: jllibre@mat.uab.cat