GLOBAL INJECTIVITY OF POLYNOMIAL MAPS VIA VECTOR FIELDS

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Abstract. This paper deals with the Real Jacobian Problem (RJP) and the Markus Yamabe Conjecture (MYC) for polynomial vector fields. We prove injectivity for a big subclass of vector fields. Concerning the MYC we get some partial results also for a big subclass of polynomial vector fields.

1. Introduction and statement of the results

Given a differentiable map $X: \mathbb{R}^n \longrightarrow \mathbb{R}^n$, denote by JX(x) the jacobian matrix of X at the point $x \in \mathbb{R}^n$. The Inverse Mapping Theorem implies that if $\det JX(x) \neq 0$ at x then X is a local diffeomorphism at x.

We denote by \mathcal{X}_m the set of all polynomial maps $X = (P^1, \ldots, P^n)$: $\mathbb{R}^n \longrightarrow \mathbb{R}^n$ such that $\deg(P^i) \le m$. By identifying \mathcal{X}_m with \mathbb{R}^M , where M is the number of all coefficients of P^1, \ldots, P^n , we endow \mathcal{X}_m with the so called coefficient topology. Notice that each $X \in \mathcal{X}_m$ can be considered as a polynomial map or as a vector field.

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