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SOME APPLICATIONS OF THE EULER-JACOBI FORMULA TO DIFFERENTIAL EQUATIONS

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ABSTRACT. The Euler-Jacobi formula gives an algebraic relation between the critical points of a vector field and their indices. Using this formula we obtain an upper bound for the number of centers that a planar polynomial differential equation can have and study the distribution of the critical points for planar quadratic and cubic differential equations.

1. STATEMENT OF MAIN RESULTS

In this paper we apply the Euler-Jacobi formula to study two different kinds of problems in the qualitative theory of ordinary differential equations: the number of centers for planar polynomial differential equations and the relationship between the indices of critical points and their distribution.

This formula will be stated in the following section. Here we state the main results in each of the subjects.

A. Centers for planar polynomial differential equations. We denote by $\mathscr{X}_{n,m}$ the set of all polynomial vector fields $X = (P, Q)$ such that the degrees of P and Q are n and m, respectively. Without loss of generality we can assume that $n \geq m$. We denote by $E(\cdot)$ the integer part function.

Our aim is to study the number of centers that $X \in \mathscr{X}_{n,m}$ can have. Define

$$C_{n,m} = \text{maximum number of centers for } X \in \mathscr{X}_{n,m},$$
$$P_{n,m} = \text{maximum number of points with index } +1 \text{ for } X \in \mathscr{X}_{n,m},$$
$$N_{n,m} = \text{maximum number of points with index } -1 \text{ for } X \in \mathscr{X}_{n,m}.$$

It is obvious that $C_{n,m} \leq P_{n,m}$. The numbers $P_{n,m}$ are studied in [K, CL]. We have that

$$P_{n,m} = \begin{cases} \dfrac{(n+1)m}{2} & \text{if } n \equiv m \pmod 2, \\ \dfrac{nm}{2} & \text{if } n \not\equiv m \pmod 2. \end{cases}$$

Our main result can be stated as follows:

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