THE GENESIS OF MARKUS YAMABE COUNTEREXAMPLES*

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ABSTRACT

In a recent paper (see [3]) it is presented a polynomial counterexample to the Markus Yamabe Conjecture in dimension $n \geq 3$. In the present work we explain the ideas to obtain this counterexample and we give a more general family of them. We also construct some polynomial maps which give a negative answer to the discrete version of the Markus Yamabe Conjecture.

1. Introduction

Let $F : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ be a C^1 map and consider the differential system

$$\dot{x} = F(x). \tag{1}$$

Assume that p is a critical point of (1), i. e., $F(p) = 0$. We say that p is a global attractor of the continuous dynamical system (1) if $\phi(t, x)$ tends to p as t tends to infinity for each $x \in \mathbb{R}^n$, where $\phi(t, x)$ is the solution of (1) with $\phi(0, x) = x$.

The next conjecture was explicitely stated by Markus and Yamabe (see [9]) in 1960.

MYC(n) (Markus-Yamabe Conjecture) Let F be a C^1 vector field defined on $\mathbb{R}^n$ such that for any $x \in \mathbb{R}^n$, the jacobian of F at x has all its

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