PERGAMON

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Nonlinear Analysis

The discrete Markus–Yamabe problem[1]

Anna Cima[a], Armengol Gasull[b], Francesc Mañosas[b,*]

[a] *Departament de Matemàtica Aplicada II, E. T. S. d'Enginyers Industrials de Terrassa, Universitat Politècnica de Catalunya, Colom, 11 08222 Terrassa, Barcelona, Spain*

[b] *Departament de Matemàtiques, Edifici C, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Spain*

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1. Statement of the problem

Let $F:\mathbf{R}^n \to \mathbf{R}^n$ be a $\mathscr{C}^1$ map and consider the differential system

$$\dot{x} = F(x). \tag{1}$$

Assume that p is a critical point of Eq. (1), i.e., $F(p)=0$. We say that p is a global attractor of the continuous dynamical system induced by Eq. (1) if $\phi(t,x)$ tends to p as t tends to infinity for each $x \in \mathbf{R}^n$, where $\phi(t,x)$ is the solution of Eq. (1) with $\phi(0,x)=x$.

The next conjecture was explicitly stated by Markus and Yamabe (see [15]) in 1960.

MYC (n) (Markus–Yamabe Conjecture). *Let F be a $\mathscr{C}^1$ map from $\mathbf{R}^n$ to itself such that for any $x \in \mathbf{R}^n$, the Jacobian of F at x has all its eigenvalues with negative real part. If $F(p)=0$, then p is a global attractor of $\dot{x}=F(x)$.*

This conjecture was proved for planar polynomial maps in 1988 [16] and for planar $\mathscr{C}^1$ maps in 1993 [9, 11] and in 1994 see [10]. In [1, 3] there are examples of smooth vector fields of $\mathbf{R}^n$, $n \geq 4$ satisfying the hypothesis of the Conjecture which have a periodic orbit and in [4] there is an example of a polynomial vector fields of $\mathbf{R}^n$, $n \geq 3$ satisfying the same hypothesis which has some orbits that scape at infinity. Therefore

* Corresponding author.

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