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Period Function for a Class of Hamiltonian Systems[1]

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This paper studies the period function of the class of Hamiltonian systems $\dot{x} = -H_y$, $\dot{y} = H_x$ where $H(x, y)$ has the special form $H(x, y) = F(x) + G(y)$ and the origin is a non-degenerate center. More concretely, if $T(h)$ denotes the period of the periodic orbit contained in $H(x, y) = h$ we solve the inverse problem of characterizing all systems with a given function $T(h)$. We also characterize the limiting behaviour of T at infinity when the origin is a global center and apply this result to prove, among other results, that there are no nonlinear polynomial isochronous centers in this family. © 2000 Academic Press

Key Words: Hamiltonian system; inverse problem; period function; isochronicity.

1. INTRODUCTION AND MAIN RESULTS

This paper deals with the class of Hamiltonian systems $\dot{x} = -H_y$, $\dot{y} = H_x$ where $H(x, y)$ has the special form

$$H(x, y) = F(x) + G(y) \tag{1}$$

with $F(x)$ and $G(y)$ analytic functions at 0, such that they have a non-degenerate minimum at $x = 0$ and $y = 0$, respectively, and $F(0) = G(0) = 0$. This last condition says that system

$$\begin{aligned} \dot{x} &= -g(y) \\ \dot{y} &= f(x) \end{aligned} \tag{2}$$

with $F'(x) = f(x)$ and $G'(y) = g(y)$ has a non-degenerate center at $(0, 0)$ and their solutions are contained in the level curves $H(x, y) = h$. We will denote by $\mathscr{P}$ the period annulus of the center, i.e., $\mathscr{P}$ is the largest neighborhood of $(0, 0)$ which is full of periodic orbits. We will say that $(0, 0)$ is a *global center* if $\mathscr{P} = \mathbb{R}^2$. It is easy to see that the orbits can be parameterized by h and give rise to a function $T(h)$ which is also analytic at 0 and satisfies $T(0) > 0$. The main goal of this paper is to study the so-called inverse problem for the period function $T(h)$. The problem is to

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