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Global periodicity and complete integrability of discrete dynamical systems

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Consider the discrete dynamical system generated by a map F. It is said that it is globally periodic if there exists a natural number p such that $F^p(x)=x$ for all x in the phase space. On the other hand, it is called completely integrable if it has as many functionally independent first integrals as the dimension of the phase space. In this paper, we relate both concepts. We also give a large list of globally periodic dynamical systems together with a complete set of their first integrals, emphasizing the ones coming from difference equations.

Keywords: Globally periodic discrete dynamical system; First integrals; Completely integrable systems; Difference equations; Invariants for difference equations

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1. Introduction and main result

Let $F:\mathcal{U}\subseteq\mathbb{K}^k\to\mathcal{U}$, be a map where $\mathcal{U}$ is an open set of $\mathbb{K}^k$ and $\mathbb{K}$ denotes either $\mathbb{R}$ or $\mathbb{C}$. This paper is concerned with some features of the dynamical system generated by F. We start by recalling some definitions.

We say that a map F is globally periodic if there exists some $p\in\mathbb{N}$ such that $F^p(x)=x$ for all $x\in\mathcal{U}$, where

$$F^p=\overbrace{F\circ\cdots\circ F}^{p\text{ times}}.$$

In particular, notice that globally periodic maps have to be bijective and $F^{-1}=F^{p-1}$. This name comes from the fact that if F is globally periodic, then the discrete dynamical system (DDS) generated by F has all its orbits as periodic orbits. The study of globally periodic maps is a current subject of interest of the dynamical systems community, see for instance Refs. [1-5,7,10,21,26]. Indeed, the functional equation $F^p=\mathrm{Id}$ is also known in the literature as Babbage equation, see Ref. [20] and it is one of the oldest iterative functional equations ever discussed.

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