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Journal of Differential Equations

J. Differential Equations 244 (2008) 630-648

www.elsevier.com/locate/jde

Studying discrete dynamical systems through differential equations

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Received 5 February 2007; revised 8 October 2007

Abstract

In this paper we consider dynamical systems generated by a diffeomorphism F defined on \mathcal{U} an open subset of \mathbb{R}^n , and give conditions over F which imply that their dynamics can be understood by studying the flow of an associated differential equation, $\dot{x} = X(x)$, also defined on \mathcal{U} . In particular the case where F has n-1 functionally independent first integrals is considered. In this case X is constructed by imposing that it shares with F the same set of first integrals and that the functional equation $\mu(F(x)) = \det(DF(x))\mu(x)$, $x \in \mathcal{U}$, has some non-zero solution, μ . Several examples for n = 2, 3 are presented, most of them coming from several well-known difference equations.

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MSC: 37C05; 37C27; 37E10; 39A20

Keywords: Conjugation of flows; Lie symmetries; Integrable vector fields; Integrable mappings; Difference equations

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0022-0396/\$ – see front matter $\hfill \ensuremath{\mathbb{C}}$ 2007 Elsevier Inc. All rights reserved. doi:10.1016/j.jde.2007.10.013

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