IOP PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND THEORETICAL

J. Phys. A: Math. Theor. **41** (2008) 285205 (18pp) doi:10.1088/1751-8113/41/28/285205

Some properties of the *k*-dimensional Lyness' map

Anna Cima[1], **Armengol Gasull**[1] **and Víctor Mañosa**[2]

[1] Dept. de Matemàtiques, Facultat de Ciències, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Spain
[2] Dept. de Matemàtica Aplicada III (MA3), Control, Dynamics and Applications Group (CoDALab) Universitat Politècnica de Catalunya (UPC) Colom 1, 08222 Terrassa, Spain

E-mail: cima@mat.uab.cat, gasull@mat.uab.cat and victor.manosa@upc.edu

Received 28 January 2008, in final form 20 May 2008
Published 19 June 2008
Online at stacks.iop.org/JPhysA/41/285205

Abstract

This paper is devoted to study some properties of the k-dimensional Lyness' map $F(x_1,\ldots,x_k)=\left(x_2,\ldots,x_k,\left(a+\sum_{i=2}^k x_i\right)/x_1\right)$. Our main result presents a rational vector field that gives a Lie symmetry for F. This vector field is used, for $k\leqslant 5$, to give information about the nature of the invariant sets under F. When k is odd, we also present a new (as far as we know) first integral for $F\circ F$ which allows us to deduce in a very simple way several properties of the dynamical system generated by F. In particular for this case we prove that, except on a given codimension one algebraic set, none of the positive initial conditions can be a periodic point of odd period.

PACS numbers: 02.30.Ik, 05.45.−a
Mathematics Subject Classification: 39A20, 37E35

(Some figures in this article are in colour only in the electronic version)

1. Introduction and main results

Discrete integrable systems are the focus of current intensive research (see [14, 18, 23, 26, 34, 35] and references therein) since they appear as fundamental mathematical tools in numerical analysis and in some areas of physics and theoretical biology such as statistical mechanics [28, 36], discrete quantum theory [5, 6], solitons in cellular automata [22, 27, 33] and population dynamics [32] among other topics.

There are some well-known planar integrable maps like the Lyness and McMillan ones, generalized by the celebrated QRT maps [29, 30], and recently a list of third-order integrable difference equation (including the third-order Lyness one) has attracted the researchers attention [18, 26, 31]. In this context, the second- and the third-order Lyness' difference equations

$$y_{n+2}=\frac{a+y_{n+1}}{y_n}\qquad\text{and}\qquad y_{n+3}=\frac{a+y_{n+1}+y_{n+2}}{y_n},\qquad\text{with}\quad a\geqslant 0$$