On the global asymptotic stability of difference equations satisfying a Markus-Yamabe condition

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Abstract

We prove a global asymptotic stability result for maps coming from n-th order difference equation and satisfying a Markus-Yamabe type condition. We also show that this result is sharp.

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1 Introduction and main results

Let $F : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ be a $\mathcal{C}^1$ map and consider the discrete dynamical system

$$\mathbf{x}_{k+1} = F(\mathbf{x}_k). \tag{1}$$

LaSalle in [13] gave some possible generalizations of the sufficient conditions for global asymptotic stability (GAS) of system (1) for $n = 1$. To state some of these conditions we first introduce some notations.

Let $A = (a_{ij})$ be a $n \times n$ matrix. We denote by $\sigma(A)$ the spectrum of A, i.e., the set of all the eigenvalues of A, by $\rho(A)$ its spectral radius, $\rho(A) = \max_{\{\lambda \in \sigma(A)\}} |\lambda|$. If A is real valued, then we write $|A| = (|a_{ij}|)$. Finally, we denote by $DF(\mathbf{x}) = (D_j(F_i)(\mathbf{x}))$ the Jacobian matrix of F at $\mathbf{x} = (x_1, x_2, \ldots, x_n) \in \mathbb{R}^n$.

The LaSalle's conditions dealing with DF are:

(I) For all $\mathbf{x} \in \mathbb{R}^n$, $\rho(DF(\mathbf{x})) < 1$.

(II) For all $\mathbf{x} \in \mathbb{R}^n$, $\rho(|DF(\mathbf{x})|) < 1$.