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NONAUTONOMOUS TWO-PERIODIC GUMOVSKI–MIRA DIFFERENCE EQUATIONS

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We consider two types of nonautonomous two-periodic Gumovski–Mira difference equations. We show that while the corresponding autonomous recurrences are conjugated, the behavior of the sequences generated by the two-periodic ones differ dramatically: in one case the behavior of the sequences is simple (*integrable*) and in the other case it is much more complicated (*chaotic*). We also present a global study of the integrable case that includes which periods appear for the recurrence.

Keywords: Integrable and chaotic difference equations and maps; rational difference equations with periodic coefficients; perturbed twist maps.

1. Introduction

Autonomous difference equations are a classical tool for the modeling of ecological systems, see for instance [Hoppensteadt, 1982; Murray, 1993; Thieme, 2003]. One of the modifications applied in the models in order to adapt them to more realistic situations consists of converting the recurrences into nonautonomous ones, changing one of the constant parameters by a periodic cycle (as in [Cushing, 1998; Cushing & Henson, 2001, 2002; Elaydi & Sacker, 2005, 2006; Sacker & von Bremen, 2010] and their references). In this situation, it is said that the model takes into account seasonality. For instance, a parameter taking values in a cycle of period four could model an ecological situation that has different features during spring, summer, autumn or winter.

In this paper, we will consider a very simple autonomous recurrence, which depends on a unique parameter and another one which is conjugated to this one. Then we will show that changing the constant parameter by a two cycle has a completely different effect in both cases. Indeed in one of them, the recurrence has an invariant while in the second one the recurrence appears to be a chaotic one. This phenomenon is very surprising and shows that this procedure used in modeling of changing the constant parameters by periodic cycles is quite delicate.