J. Differential Equations 255 (2013) 339–350

Contents lists available at SciVerse ScienceDirect

Journal of Differential Equations

www.elsevier.com/locate/jde

An explicit bound of the number of vanishing double moments forcing composition

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ARTICLE INFO

Article history:
Received 23 November 2012
Available online 15 April 2013

MSC:
39C25
37C27

Keywords:
Trigonometric Abel equation
Double moments
Composition pair

ABSTRACT

We give two new characterizations of pairs of polynomials or trigonometric polynomials that form a composition pair. One of them proves that the cancellation of a given number of double moments implies that they form a composition pair. This number only depends on the maximum degree of both polynomials. This is the first time that composition is characterized in terms of the cancellation of an explicit number of double moments. Our results allow to recognize the composition centers for polynomial and trigonometric Abel differential equations.

1. Introduction and main results

Abel equations of the form

$$\dot{r} = \frac{dr}{ds} = A(s)r^3 + B(s)r^2, \tag{1}$$

with A and B either polynomials or trigonometric polynomials are a subject of increasing interest; see [6–11,16,25]. One of the main reasons is their relation with the center-focus problem and the second part of the Hilbert Sixteenth problem. Both questions deal with the number of periodic orbits of planar polynomial systems; see [2,4,15,17,21,24]. In particular, given $a < b$, the center-focus problem in this setting reduces to find conditions on A and B such that all the solutions $r = r(s, r_0)$, with initial condition $r(a, r_0) = r_0$ and $|r_0|$ small enough, satisfy $r(a, r_0) = r(b, r_0)$. When this happens it is said that the Abel equation has a *center* at the origin, $r = 0$. The case where A and B are trigonometric

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0022-0396/$ – see front matter
http://dx.doi.org/10.1016/j.jde.2013.04.009