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Integrability and non-integrability of periodic non-autonomous Lyness recurrences

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This paper studies non-autonomous Lyness-type recurrences of the form $x_{n+2} = (a_n + x_{n+1})/x_n$, where $\{a_n\}$ is a k-periodic sequence of positive numbers with primitive period k. We show that for the cases $k \in \{1, 2, 3, 6\}$, the behaviour of the sequence $\{x_n\}$ is simple (integrable), while for the remaining cases satisfying this behaviour can be much more complicated (chaotic). We also show that the cases where k is a multiple of 5 present some different features.

Keywords: Integrability and non-integrability of discrete systems; numerical chaos; periodic difference equations; QRT maps; rational and meromorphic first integrals

1. Introduction and main results

The dynamical study of the Lyness difference equation [1–4] and its generalizations to higher order Lyness-type equations, [5–8] or to difference equations with periodic coefficients, [9–14] has been the focus of an active research activity in the last two decades. In more recent dates, Lyness-type equations have also been approached using different points of view: from algebraic geometry [15–17] to the theory of discrete integrable systems [14,18–21].

This paper deals with the problem of the integrability and non-integrability of non-autonomous planar Lyness difference equations of the form

$$x_{n+2} = \frac{a_n + x_{n+1}}{x_n}, \tag{1}$$

where $\{a_n\}$ is a cycle of k positive numbers, that is, $a_{n+k} = a_n$ for all $n \in \mathbb{N}$, k being the primitive period and we consider positive initial conditions x_1 and x_2. As we will see, the behaviour of the sequences $\{x_n\}$ can be essentially different according to whether $k \in \{1, 2, 3, 6\}$, k is a multiple of 5 or not.

In this section, we summarize our main results on Equation (1) in terms of k. We also give an account of the tools that we have developed for this study that we believe might be interesting by themselves. We start by introducing the notations and definitions used in the paper.

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