Journal of Difference Equations and Applications, 2014
Vol. 20, No. 3, 423–437, http://dx.doi.org/10.1080/10236198.2013.852187

Basin of attraction of triangular maps with applications

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(*Received 25 July 2013; accepted 1 October 2013*)

We consider planar triangular maps $x_{n+1}=f_0(u_n)+f_1(u_n)x_n, u_{n+1}=\phi(u_n)$. These maps preserve the fibration of the plane given by $\mathcal{F}=\{u=c, c\in R\}$. We assume that there exists an invariant attracting fibre $\{u=u_*\}$ for the dynamical system generated by ϕ, and we study the limit dynamics of those points in the basin of attraction of this invariant fibre, assuming that either it contains a global attractor or it is filled by fixed or two-periodic points. We apply our results to several examples.

Keywords: attractors; difference equations; discrete dynamical systems; triangular maps; periodic solutions; quasi-homogeneous maps

MSC (2010) Classification: 39A10; 39A11; 39A20

1. Introduction

In this paper we consider triangular systems of the form

$$\begin{cases} x_{n+1}=f_0(u_n)+f_1(u_n)x_n, \\ u_{n+1}=\phi(u_n), \end{cases} \tag{1}$$

where $\{x_n\}$ and $\{u_n\}$ are real sequences and f_0, f_1 and ϕ are continuous functions.

Observe that system (1) preserves the fibration of the plane given by $\mathcal{F}=\{u=c, c\in R\}$, that is it sends fibres of $\mathcal{F}$ to others. We assume that there exists a point $u=u_*$ which is a local stable attractor of the subsystem $u_{n+1}=\phi(u_n)$. In this case, we say that system (1) has a *local attractive fibre* $\{u=u_*\}$. Our objective is to know whether the asymptotic dynamics of the orbits corresponding to points in the *basin of attraction* of the *limit fibre* $\{u=u_*\}$ is characterized by the dynamics on this fibre, the *limit dynamics*. In all the cases, we assume that the limit dynamics is very simple, that is: either (A) the fibre $\{u=u_*\}$ contains a global attractor, see Proposition 2; (B) the fibre is filled by fixed points, Theorem 6(a) or (C) it is filled by two-periodic orbits, Theorem 6(b). Observe that there is no need to consider the case in which there is a global repellor on the fibre $\{u=u_*\}$, since in this situation it is clear that many orbits with initial conditions in the basin of attraction of this fibre are unbounded.

The paper is structured as follows. In Section 2 we present the main results (Proposition 2, Theorem 6 and Corollary 7) together with some motivating examples.

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