Parrondo's dynamic paradox for the stability of non-hyperbolic fixed points

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Abstract

We show that for periodic non-autonomous discrete dynamical systems, even when a common fixed point for each of the autonomous associated dynamical systems is repeller, this fixed point can became a local attractor for the whole system, giving rise to a Parrondo's dynamic type paradox.

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1 Introduction and main results

The study of periodic discrete dynamical systems is a classical topic that has attracted the researcher's interest in the last years, among other reasons, because they are good models for describing the dynamics of biological systems under periodic fluctuations whether due to external disturbances or effects of seasonality, see [4, 15, 16, 17, 25, 26, 27] and the references therein.

These k-periodic systems can be written as

$$x_{n+1} = f_{n+1}(x_n), \tag{1}$$

with initial condition x_0, and a set of maps $\{f_m\}_{m\in\mathbb{N}}$ such that $f_m = f_\ell$ if $m \equiv \ell \,(\bmod\, k)$. For short, the set $\{f_1, \ldots, f_k\}$ will be called *periodic set.* We also will assume that all $f_m : \mathcal{U} \subset \mathbb{R}^n \to \mathcal{U}$ being $\mathcal{U}$ an open set of $\mathbb{R}^n$.