



On the number of polynomial solutions of Bernoulli and Abel polynomial differential equations

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Abstract

In this paper we determine the maximum number of polynomial solutions of Bernoulli differential equations and of some integrable polynomial Abel differential equations. As far as we know, the tools used to prove our results have not been utilized before for studying this type of questions. We show that the addressed problems can be reduced to know the number of polynomial solutions of a related polynomial equation of arbitrary degree. Then we approach to these equations either applying several tools developed to study extended Fermat problems for polynomial equations, or reducing the question to the computation of the genus of some associated planar algebraic curves.

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1. Introduction

In this work we investigate the number of polynomial solutions of some differential equations of type

$$q(t) \dot{x} = p_n(t) x^n + p_{n-1}(t) x^{n-1} + \cdots + p_1(t) x + p_0(t) \quad (1)$$

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