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Bifurcation of 2-periodic orbits from non-hyperbolic fixed points ☆

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ABSTRACT

We introduce the concept of 2-cyclicity for families of one-dimensional maps with a non-hyperbolic fixed point by analogy to the cyclicity for families of planar vector fields with a weak focus. This new concept is useful in order to study the number of 2-periodic orbits that can bifurcate from the fixed point. As an application we study the 2-cyclicity of some natural families of polynomial maps.

1. Introduction

The cyclicity of a family of vector fields having a weak focus or a center is a well known concept in the theory of planar vector fields and the problems surrounding the second part of the Hilbert's 16th problem [11,16]. A grosso modo the cyclicity expresses the maximum number of small amplitude limit cycles that can effectively bifurcate from the singular point by varying the parameters in the family of considered vector fields.

This cyclicity is given by the number of fixed points near the critical point of a family of orientation preserving maps (the so called return maps) with a non-hyperbolic fixed point. As we will see, the cyclicity also can be seen as the number of 2-periodic orbits of a related family of orientation reversing maps (the half-return maps), see for instance [4] or Section 4. Recall that given a map $f : \mathbb{R} \to \mathbb{R}$, a 2-periodic orbit is a set $\{x, y\}$ such that $f(x) = y, f(y) = x$ and $x \neq y$.

Hence it is natural, in the discrete setting, to study the bifurcation of 2-periodic orbits from non-hyperbolic fixed points of orientation reversing one-dimensional analytic diffeomorphisms of the form

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