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Around some extensions of Casas-Alvero conjecture for non-polynomial functions

A. Cima, A. Gasull, F. Mañosas

Departament de Matemàtiques, Universitat Autònoma de Barcelona
Barcelona, Spain

cima@mat.uab.cat , gasull@mat.uab.cat , manyosas@mat.uab.cat

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Abstract: We show that two natural extensions of the real Casas-Alvero conjecture in the non-polynomial setting do not hold.

Key words: polynomial, Casas-Alvero conjecture, zeroes of functions.

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1. Introduction

The Casas-Alvero conjecture affirms that if a complex polynomial P of degree $n>1$ shares roots with all its derivatives, $P^{(k)}$, $k=1,2,\ldots,n-1$, then there exist two complex numbers, a and $b\neq 0$, such that $P(z)=b(z-a)^n$. Notice that, in principle, the common root between P and each $P^{(k)}$ might depend on k. Casas-Alvero arrived to this problem at the turn of this century, when he was working in his paper [1] trying to obtain an irreducibility criterion for two variable power series with complex coefficients. See [2] for an explanation of the problem in his own words.

Although several authors have got partial answers, to the best of our knowledge the conjecture remains open. For $n\leq 4$ the conjecture is a simple consequence of the wonderful Gauss-Lucas Theorem ([6]). In 2006 it was proved in [5], by using Maple, that it is true for $n\leq 8$. Afterwards in [6, 7] it was proved that it holds when n is p^m, $2p^m$, $3p^m$ or $4p^m$, for some prime number p and $m\in\mathbb{N}$. The first cases left open are those where $n=24$, 28 or 30. See again [6] for a very interesting survey or [3, 8] for some recent contributions on this question.

Adding the hypotheses that P is a real polynomial and all its n roots, taking into account their multiplicities, are real, the conjecture has a real

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