Different Approaches to the Global Periodicity Problem

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Abstract Let F be a real or complex n-dimensional map. It is said that F is *g*lobally periodic if there exists some $p \in \mathbb{N}^+$ such that $F^p(x) = x$ for all x, where $F^k = F \circ F^{k-1}$, $k \geq 2$. The minimal p satisfying this property is called the *p*eriod of F. Given a m-dimensional parametric family of maps, say F_λ, a problem of current interest is to determine all the values of λ such that F_λ is globally periodic, together with their corresponding periods. The aim of this paper is to show some techniques that we use to face this question, as well as some recent results that we have obtained. We will focus on proving the equivalence of the problem with the complete integrability of the dynamical system induced by the map F, and related issues; on the use of the local linearization given by the Bochner Theorem; and on the use the Normal Form theory. We also present some open questions in this setting.

Keywords Globally periodic maps · Integrable discrete systems · Lie Symmetries · Linearizations · Periodic difference equations · Reversible maps

1 Introduction

This paper deals with *globally periodic* maps and difference equations. A map $F : \mathscr{U} \to \mathscr{U}$, defined on an open set $\mathscr{U} \subseteq \mathbb{K}^n$ (where $\mathbb{K}$ is either $\mathbb{R}$ or $\mathbb{C}$) is *globally*

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L. Alsedà i Soler et al. (eds.), *Difference Equations, Discrete Dynamical Systems and Applications*, Springer Proceedings in Mathematics & Statistics 180, DOI 10.1007/978-3-662-52927-0_7