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Research paper

# Pointwise periodic maps with quantized first integrals

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ABSTRACT

We describe the global dynamics of some pointwise periodic piecewise linear maps in the plane that exhibit interesting dynamic features. For each of these maps we find a first integral. For these integrals the set of values are discrete, thus quantized. Furthermore, the level sets are bounded sets whose interior is formed by a finite number of open tiles of certain regular or uniform tessellations. The action of the maps on each invariant set of tiles is described geometrically.


## 1. Introduction

A *pointwise periodic map* is a bijective self-map in a topological space such that each point is periodic. A *periodic map* is a bijective self-map in a topological space such that some iterated of the map is the identity. For a periodic map $F : X \longrightarrow X$ the minimum natural number $p$ satisfying $F^p = \mathrm{Id}$ is called *the period* of $F$. Notice that a pointwise periodic map satisfying that the period of the points has an upper bound is periodic and its period is the least common multiple of the periods of the elements of the space.

A classical result of Montgomery establishes that any *pointwise periodic homeomorphism* in an Euclidean space is *periodic*, [1]. Non-periodic but pointwise periodic bijective maps do exist when the continuity assumption is relaxed, see [2] for instance. In the series of papers [3–5], the authors introduce three explicit examples of pointwise periodic maps that are not periodic. The examples given by these authors in the above mentioned references belong to the family of piecewise affine maps with a line of discontinuity:

$$G(x, y) = (y, -x - \rho y + \mathrm{sign}(y)), \quad \text{where} \quad \mathrm{sign}(y) = \begin{cases} +1, & \text{if } y \geq 0; \\ -1, & \text{otherwise,} \end{cases} \tag{1}$$

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