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Linearization of planar involutions in $\mathcal{C}^1$

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Abstract The celebrated Kerékjártó Theorem asserts that planar continuous periodic maps can be continuously linearized. We prove that $\mathcal{C}^1$-planar involutions can be $\mathcal{C}^1$-linearized.

Keywords Kerékjártó theorem · Periodic maps · Linearization · Involution

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1 Introduction and statement of the main result

A map $F : \mathbb{R}^n \to \mathbb{R}^n$ is called *m-periodic* if $F^m = \text{Id}$, where $F^m = F \circ F^{m-1}$, and m is the smallest positive natural number with this property. When $m = 2$, then it is said that F is an *involution*.

When there exists a $\mathcal{C}^k$-diffeomorphism $\psi : \mathbb{R}^n \to \mathbb{R}^n$, such that $\psi \circ F \circ \psi^{-1}$ is a linear map, then it is said that F is $\mathcal{C}^k$-*linearizable*. In this case, the map ψ is called a *linearization* of F. This property is very important because it is not difficult to describe the dynamics of the discrete dynamical system generated by linearizable maps. For instance, planar m-periodic linearizable maps behave as planar m-periodic linear maps: They are either symmetries with respect to a "line" or "rotations."

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