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On the number of critical periods for planar polynomial systems

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Abstract

In this paper we get some lower bounds for the number of critical periods of families of centers which are perturbations of the linear one. We give a method which lets us prove that there are planar polynomial centers of degree ℓ with at least $2[(\ell-2)/2]$ critical periods as well as study concrete families of potential, reversible and Liénard centers. This last case is studied in more detail and we prove that the number of critical periods obtained with our approach does not increases with the order of the perturbation.

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Keywords: Period function; Critical periods; Perturbations; Potential systems; Reversible centers; Hamiltonian centers; Liénard centers

1. Introduction

Consider the set $\mathcal{V}_\ell$ of all the polynomial vector fields of the form

$$\begin{aligned} \dot{x} &= -y + P(x, y) = -y + \sum_{n=2}^{\ell} P_n(x, y), \\ \dot{y} &= x + Q(x, y) = x + \sum_{n=2}^{\ell} Q_n(x, y), \end{aligned} \tag{1}$$

having a center at the origin, where P_n and Q_n are homogeneous polynomials of degree n. Given a vector field $X \in \mathcal{V}_\ell$ let $\mathcal{P}$ be the period annulus of the center, *i.e.* the open subset of the phase plane formed by all the periodic orbits of X surrounding the origin. The period function $T : \mathcal{P} \longrightarrow \mathbb{R}^+$ associates with any point $(x, y) \in \mathcal{P}$ the period of the periodic orbit passing through (x, y). Since all the points belonging to the same periodic orbit γ have the same period we may denote by $T(\gamma)$ the period of the periodic orbit. We say that T is an increasing (resp. decreasing) function if for any couple of periodic orbits γ_0 and γ_1 in $\mathcal{P}$ with γ_0 contained in the region surrounded by γ_1, we have that $T(\gamma_1) - T(\gamma_0) > 0$ (resp. $T(\gamma_1) - T(\gamma_0) < 0$). The local maximum or minimum of the period function are called critical periods.

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