Area Preserving Flows on Compact Connected Surfaces

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Abstract. Let S be an orientable compact connected surface without boundary and let X be a C^2 vector field such that its flow is area preserving on S. Assume that X has finitely many singular points, satisfies a Lojasiewicz condition at all its singular points and has singular points if S is the 2-dimensional torus. Then every singular point is either a center or it has a neighbourhood which splits into a finite union of an even number of hyperbolic sectors and all orbits of X are closed except finitely many of them which are either singular points or separatrices joining singular points. The same result is true for measure preserving flows on S. In this case it is not necessary that S be orientable.

1 Introduction and Statement of the Main Results

Let S be a C^∞ compact connected surface without boundary endowed with a metric. Then S always has a C^1 isometric imbedding in $\mathbf{R}^3$. Moreover S always has a C^∞ isometric imbedding in some $\mathbf{R}^n$, see Nash (1956). Therefore without loss of generality we can assume that the surface S is imbedded in some $\mathbf{R}^n$ and that its metric is the induced metric by the Euclidean metric of $\mathbf{R}^n$.

A C^2 *flow* on S is a C^2 map $\varphi : \mathbf{R} \times S \to S$ such that

(1) $\varphi(0,p) = p$ for all $p \in S$;

(2) $\varphi(t,\varphi(s,p)) = \varphi(t+s,p)$ for all $p \in S$ and all $t, s \in \mathbf{R}$.

Sometimes we denote $\varphi(t,\cdot)$ by φ_t. The pair (S,φ_t) is called a *dynamical system*.

We denote by T_pS the *tangent plane* to S at p and let TS be the *tangent bundle* to S. A vector field $X : S \to TS$ is a C^2 map such that $X(p) \in T_pS$ for all $p \in S$.

Let X be a C^2 vector field on S. We say that X is *area preserving* if the flow defined by X preserves the area defined by the metric of S.

In order to determine all possible local phase portraits at isolated singular points of a C^2 area preserving vector field on the surface S we assume that