Applicable Analysis
Vol. 87, No. 2, February 2008, 149–164

Taylor & Francis
Taylor & Francis Group

Limit cycles of some polynomial differential systems in dimension 2, 3 and 4, via averaging theory

ANNA CIMA†, JAUME LLIBRE*† and MARCO ANTONIO TEIXEIRA‡

†Departament de Matemàtiques, Universitat Autònoma de Barcelona,
08193 Bellaterra, Barcelona, Spain
‡Departamento de Matemática, Universidade Estadual de Campinas,
Caixa Postal 6065, 13083-970, Campinas, S.P., Brazil

Communicated by P. Broadbridge

(*Received 22 December 2006; revised 20 June 2007; in final form 6 July 2007*)

In the qualitative study of a differential system it is important to know its limit cycles and their stability. Here through two relevant applications, we show how to study the existence of limit cycles and their stability using the averaging theory. The first application is a 4-dimensional system which is a model arising in synchronization phenomena. Under the natural assumptions of this problem, we can prove the existence of a stable limit cycle. It is known that perturbing the linear center $\dot{x} = -y$, $\dot{y} = x$, up to first order by a family of polynomial differential systems of degree n in $\mathbb{R}^2$, there are perturbed systems with $(n-1)/2$ limit cycles if n is odd, and $(n-2)/2$ limit cycles if n is even. The second application consists in extending this classical result to dimension 3. More precisely, perturbing the system $\dot{x} = -y$, $\dot{y} = x$, $\dot{z} = 0$, up to first order by a family of polynomial differential systems of degree n in $\mathbb{R}^3$, we can obtain at most $n(n-1)/2$ limit cycles. Moreover, there are such perturbed systems having at least $n(n-1)/2$ limit cycles.

Keywords: Limit cycle; Synchronization; Averaging method; Linear center; Polynomial differential system

1991 Mathematics Subject Classifications: 34C29; 34C25; 47H11

1. Introduction and statement of the main results

In this article we present two applications of the averaging theory for studying the existence and the kind of stability of limit cycles for polynomial differential systems in dimensions 2, 3 and 4.

*Corresponding author. Tel.: 34 93 581 13 03. Fax: 34 93 581 27 90. Email: jllibre@mat.uab.es

Applicable Analysis ISSN 0003-6811 print/ISSN 1563-504X online © 2008 Taylor & Francis
http://www.tandf.co.uk/journals
DOI: 10.1080/00036810701556136