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Hopf Bifurcation in Presence of $1:3$ Resonance

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Abstract

Assume that the linear part at a singular point p_0 of a $\mathcal{C}^4$ differential system Y_0 in $\mathbb{R}^4$ has eigenvalues $\pm\alpha i$ and $\pm\beta i$ such that $\beta/\alpha = 1/3$. In the main result of the paper we exhibit a one–parameter family of systems Y_ε for $\varepsilon \in (-\delta_0, +\delta_0)$ where is shown that the original vector field around p_0 can bifurcate in 0, 1, 2, 3 or 4 one–parameter families of periodic orbits. The tool for proving such a result is the averaging theory for non–$\mathcal{C}^1$ differentiable system. Moreover, assuming now that Y_ε is a one–parameter family of $\mathbb{Z}_2$–reversible polynomial vector fields of degree 5, we show that it can bifurcate in 0 or 2 one–parameter families of periodic orbits.

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