Real Dynamics of Integrable Birational Maps

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Abstract We begin by studying the dynamic generated by iteration of birational maps in \mathbb{R}^k with k - 1 independent rational first integrals. We prove that each level curve can be desingularized and compactified being homeomorphic to a finite union of disjoint circles and open intervals. Furthermore, the map can be extended homeomorphically in a natural way to this space. After, we focus our attention in the case that the map has a rational invariant measure and we see that in most cases the orbit of a point or it is periodic or it fulfills densely some connected components of its corresponding level set. Some applications in dimension two and three are presented.

 $\label{eq:constraint} \begin{array}{l} \textbf{Keywords} & Birational maps \cdot Circle maps \cdot Difference equations \cdot Discrete dynamical systems \cdot First integrals \cdot Integrable maps \cdot Lie-symmetries \cdot Periodic orbits \cdot Rotation numbers \end{array}$

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1 Introduction

In this paper we study the dynamic generated by iteration of birational maps F(x), $x \in \mathbb{R}^k$ having k - 1 functionally independent rational first integrals.

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