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Isochronicity for Several Classes of Hamiltonian Systems[1]

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In this paper we study isochronous centers of analytic Hamiltonian systems giving special attention to the polynomial case. We first revisit the potential systems and we show the connection between isochronicity and involutions. We then study a more general system, namely the ones associated to Hamiltonians of the form $H(x, y) = A(x) + B(x)\, y + C(x)\, y^2$. As an application we classify the cubic polynomial Hamiltonian isochronous centers and we give examples of nontrivial and nonglobal polynomial Hamiltonian isochronous centers. © 1999 Academic Press

1. INTRODUCTION

In this paper we study isochronous centers of analytic Hamiltonian systems giving special attention to the polynomial case.

The problem of characterizing isochronous centers has attracted the attention of several authors. However there are very few families of polynomial differential systems in which a complete classification of the isochronous centers has been found. Quadratic systems were classified by Loud [8] and cubic systems with homogeneous nonlinearities by Pleshkan [11]. Kukles' systems were classified in [3]. Some other results can be found in [1]. Concerning Hamiltonian systems there are also very few results. It is proved in [2] that in the potential case the unique polynomial isochronous center is the linear one. Several authors (see [3, 5, 13]) proved that there are not Hamiltonian systems with homogeneous nonlinearities having an isochronous center at the origin. Apart from few other special cases, the knowledge of polynomial systems with isochronous centers is slight. For some other results on isochronicity we refer the reader to [3, 6, 9, 10] and references there in.

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