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FLOWS WITHOUT WANDERING POINTS ON COMPACT CONNECTED SURFACES

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Abstract. Given a compact 2–dimensional manifold M we classify all continuous flows φ without wandering points on M. This classification is performed by finding finitely many pairwise disjoint open $\varphi-$invariant subsets $\{U_1, U_2, \ldots, U_n\}$ of M such that $\bigcup_{i=1}^{n} \overline{U_i} = M$ and each U_i is either a suspension of an interval exchange transformation, or a maximal open cylinder made up of closed trajectories of φ.

1. Introduction and statement of the main results

Let M be a C^∞ compact connected 2–dimensional manifold without boundary. Let $\varphi : \mathbb{R} \times M \to M$ be a *flow* (on M); that is, φ is continuous and

(i) $\varphi(0, p) = p$ for all $p \in M$;
(ii) $\varphi(t, \varphi(s, p)) = \varphi(t + s, p)$ for all $p \in M$ and all $t, s \in \mathbb{R}$.

We say that the flow $\varphi : \mathbb{R} \times M \to M$ is *without wandering points* if for every pair $(\mathcal{V}, R)$ consisting of an open subset $\mathcal{V}$ of M and a real number $R > 0$, there exists a real number $|t| > R$ such that $\varphi_t(\mathcal{V}) \cap \mathcal{V} \neq \emptyset$, where $\varphi_t(\mathcal{V}) := \{\varphi(t, p) : p \in \mathcal{V}\}$. Continuous *area-preserving* flows are examples of flows without wandering points.

The theorem below is our main result on the dynamics of continuous flows on compact connected surfaces. To state it we shall need the following definitions.

We define the *ω–limit set* (resp. *α–limit set*) of $p \in M$ as the set $\omega(p)$ (resp. $\alpha(p)$) made up of the $q \in M$ such that for some sequence of real numbers $t_n \to \infty$ (resp. $t_n \to -\infty$), $\varphi(t_n, p) \to q$.

The *positive* (resp. *negative*) *half–trajectory* or *semi–trajectory* γ_p^+ (resp. γ_p^-) through the point $p \in M$ is the set $\{\varphi(t, p) : t \geq 0\}$ (resp. $\{\varphi(t, p) : t \leq 0\}$). The *orbit* or *trajectory* γ_p through the point $p \in M$ is the set $\gamma_p^+ \cup \gamma_p^-$. The *$\omega$–limit set* (respectively *α–limit set*) of an orbit γ is the set $\omega(p)$ (respectively $\alpha(p)$) for some $p \in \gamma$.

A point p is a *fixed point* of φ if the orbit of p is the set $\{p\}$. An orbit is said to be *regular* if it is not reduced to a fixed point.

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