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Alien limit cycles in Liénard equations

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ABSTRACT

This paper aims at providing an example of a family of polynomial Liénard equations exhibiting an alien limit cycle. This limit cycle is perturbed from a 2-saddle cycle in the boundary of an annulus of periodic orbits given by a Hamiltonian vector field. The Hamiltonian represents a truncated pendulum of degree 4. In comparison to a former polynomial example, not only the equations are simpler but a lot of tedious calculations can be avoided, making the example also interesting with respect to simplicity in treatment.

1. Introduction

Periodic orbits in polynomial planar differential systems can be isolated or belong to an annulus of periodic orbits. In the isolated case they are called limit cycles.

For planar real polynomial vector fields, Hilbert's 16th problem (see [9]) is involved with the question of the existence of a finite upper bound, only depending on the degree of the vector field, of the number of limit cycles.

A way to get insight in Hilbert 16th problem is by using perturbative methods, i.e. to ask for the maximum number of limit cycles when perturbing from known situations like, for instance, from polynomial Hamiltonian systems, i.e. those described by a polynomial Hamiltonian function. The latter problem is known as the Infinitesimal Hilbert's 16th problem (see [1,2]). It deals with differential systems of the form

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