International Journal of Bifurcation and Chaos, Vol. 19, No. 3 (2009) 765–783

PHASE PORTRAITS OF THE QUADRATIC SYSTEMS WITH A POLYNOMIAL INVERSE INTEGRATING FACTOR

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Received April 1, 2008; Revised July 31, 2008

We classify the phase portraits of all planar quadratic polynomial differential systems having a polynomial inverse integrating factor.

Keywords: Polynomial inverse integrating factor; quadratic differential systems; Darboux theory of integrability.

1. Introduction and Statement of the Results

Let P and Q be two real polynomials in the variables x and y. We say that $X = (P, Q) : \mathbb{R}^2 \to \mathbb{R}^2$ is a *polynomial vector field of degree* m if the maximum of the degrees of the polynomials P and Q is m. The system

$$\dot{x} = P(x, y), \quad \dot{y} = Q(x, y), \tag{1}$$

where the dot denotes the derivative with respect to the time variable t, is the *real planar polynomial differential system* of degree m associated to X. We denote by $\mathbf{X} = P(\partial/\partial x) + Q(\partial/\partial y)$ the linear operator associated to (1).

The *inverse integrating factor* V associated to a first integral H of the differential system $\dot{x} = P(x, y), \dot{y} = Q(x, y)$ is a solution of the system

$$\frac{P}{V} = -\frac{\partial H}{\partial y}, \quad \frac{Q}{V} = \frac{\partial H}{\partial x}.$$

If for a given differential system we know H then we know V, and vice versa.

The *quadratic systems* are the polynomial real differential systems of type (1) of degree $m = 2$. Quadratic systems have been investigated intensively, and more than one thousand papers have been published about these vectors fields (see for instance [Reyn, 1989; Ye *et al.*, 1984; Ye, 1995]), but the problem of classifying all the integrable quadratic vector fields remains open. For more information on integrable differential vector fields in dimension 2, see [Chavarriga *et al.*, 1999].

The *phase portrait* of a system is the decomposition of its domain of definition as union of all its orbits. The goal of this paper is to provide the topological classification of the phase portraits of the quadratic systems having a polynomial inverse integrating factor. Such systems are studied and classified in [Coll *et al.*, preprint 2008]. We reproduce