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Polynomial inverse integrating factors for quadratic differential systems

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ABSTRACT

We characterize all the quadratic polynomial differential systems having a polynomial inverse integrating factor and provide explicit normal forms for such systems and for their associated first integrals. We also prove that these families of quadratic systems have no limit cycles.

1. Introduction and statement of the main results

Nonlinear ordinary differential equations appear in many branches of applied mathematics, physics and, in general, applied sciences. For a differential system or a vector field defined on the plane $\mathbb{R}^2$ the existence of a first integral determines completely its phase portrait. Since for such vector fields the notion of integrability is based on the existence of a first integral the following natural question arises: *Given a vector field on* $\mathbb{R}^2$, *how does one recognize whether it has a first integral?*

Let P and Q be two real polynomials in the variables x and y. We say that $X=(P,Q):\mathbb{R}^2\to\mathbb{R}^2$ is a *polynomial vector field of degree* m if the maximum of the degrees of the polynomials P and Q is m. The system

$$\dot{x}=P(x,y),\qquad \dot{y}=Q(x,y), \tag{1}$$

where the dot denotes the derivative with respect to the time variable t, is the *real planar polynomial differential system* of degree m associated with X. The polynomial vector field $X=(P,Q)$ associated with system (1) will also be denoted by $\mathbf{X}=P\partial_x+Q\partial_y$.

A C^k function $H:U\to\mathbb{R}$, with $k\geq 1$, is a *first integral* of system (1) if $\mathbf{X}H=0$ on the domain of definition of H.

An *inverse integrating factor* of (1) is a solution V of the equation $\mathbf{X}V=V\,\mathrm{div}\,(X)$. The inverse integrating factor V associated with a first integral H of the differential system $\dot{x}=P(x,y),\dot{y}=Q(x,y)$ satisfies

$$\frac{P}{V}=-\frac{\partial H}{\partial y},\qquad \frac{Q}{V}=\frac{\partial H}{\partial x}.$$

If for a given differential system we know H then we know V, and vice versa.

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