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FIRST LYAPUNOV CONSTANTS FOR NON-SMOOTH LIÉNARD DIFFERENTIAL EQUATIONS

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Abstract. By using a modification of the standard method given by Lyapunov, we compute the first seven Lyapunov constants for a non-smooth Liénard differential equation. This result suggests that all centers inside this family are reversible.

1. Introduction

The Liénard equation plays an important role, among others, in the theory of non-linear electrical circuits and in the study of periodic orbits of several predator-prey models, see for instance [4]. The theory of non-smooth differential systems becomes increasinly important in engineering and other applied sciences. The goal of this paper is to study the stability of the origin and the center-focus problem for the non-smooth Liénard equation,

$$(1)\qquad (\dot{x},\dot{y}) = \begin{cases} (-y+f^{+}(x),x) & \text{if} \quad y \geq 0, \\ (-y+f^{-}(x),x) & \text{if} \quad y \leq 0, \end{cases}$$

where $f^{\pm}(x) = \sum_{i\geq 2} a_i^{\pm} x^i$, $a_i^{\pm} \in \mathbb{R}$. For short, sometimes, we will write $a_i = a_i^{+}$ and $b_i = a_i^{-}$, for all $i \geq 2$. This problem has been solved for the smooth case (*i.e.* $f^{+} \equiv f^{-}$) in [3] and [7].

Our main result is :

Theorem A. The first Lyapunov constants for equation (1) are,

$$\begin{aligned} &V_1 = V_2 = 0, && V_5 = \tfrac{5\pi}{16}(a_5+b_5), \\ &V_3 = \tfrac{3\pi}{8}(a_3+b_3), && V_6 = \tfrac{26}{21}a_5(a_2+b_2) + \tfrac{58}{525}a_3(a_4+b_4), \\ &V_4 = \tfrac{14}{15}a_3(a_2+b_2), && V_7 = \tfrac{35\pi}{128}(a_7+b_7). \end{aligned}$$