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BIFURCATION OF LIMIT CYCLES FROM TWO FAMILIES OF CENTERS

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Abstract. We study the number of limit cycles that bifurcate from the periodic orbits of a center in two families of planar polynomial systems. One of these families has a global center. The other family is obtained by adding a straight line of critical points to the first one. The common point between both unperturbed families is that they can be integrated by using the Lyapunov polar coordinates. The study of the number of limit cycles bifurcating from the centers is done by considering the zeros of the associated Poincaré-Melnikov integrals. As a consequence of our study we provide quadratic lower bounds for the number of limit cycles surrounding a unique critical point in terms of the degree of the system.

Keywords. bifurcation, limit cycle, center, Abelian integral
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1 Introduction and Main Results

A complete study of the real planar differential systems requires, from the qualitative point of view, determining the number and nature of critical points, the separatrix structure and the number and location of closed trajectories. The third question is still the main open problem in the qualitative theory of real planar differential systems.

In the study of two-dimensional systems,

$$\dot{x} = P(x,y), \quad \dot{y} = Q(x,y), \tag{1}$$

where P and Q are real polynomials in the variables x and y, the determination of the number and position of limit cycles is known as the second part of the *Hilbert 16^{th} problem* (see [6]). Let $\mathcal{H}_n$ denote the maximum possible number of limit cycles of system (1) when P and Q are arbitrary polynomials of degree at most n. The $\mathcal{H}_n$ are known as the Hilbert numbers, and it is still an open problem whether $\mathcal{H}_n$ is finite or not for each $n \geq 2$. To know about $\mathcal{H}_n$ one strategy is to control its lower bounds in terms of n. This can be done by studying particular families of polynomial differential systems of type (1).

In this paper we consider systems of the form

$$\frac{dx}{dt} = -V(x,y)\frac{\partial H(x,y)}{\partial y} + \varepsilon P(x,y), \quad \frac{dy}{dt} = V(x,y)\frac{\partial H(x,y)}{\partial x} + \varepsilon Q(x,y), \tag{2}$$