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# ON A CRITERIUM OF GLOBAL ATTRACTION FOR DISCRETE DYNAMICAL SYSTEMS

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Abstract. Consider that the origin is a fix point of a discrete dynamical system $x^{(n+1)} = F(x^{(n)})$, defined in the whole $\mathbb{R}^m$. LaSalle, in his book of 1976, [13], proposes to study several conditions which might imply global attraction. One of his suggestions is to write $F(x) = A(x)x$, where $A(x)$ is a real $m\times m$ matrix, and to assume that all the eigenvalues of eigenvalues of $A(x)$, for all $x \in \mathbb{R}^m$, have modulus smaller than one. In the paper [4], Cima *et al.* show that, when $m \geq 2$, such hypothesis does not guarantee that the origin is a global attractor, even for polynomial maps $F$. From the observation that the decomposition of $F(x)$ as $A(x)x$ is not unique, in this paper we wonder whether LaSalle condition, for a special and canonical choice of $A$, forces the origin to be a global attractor. This canonical choice is given by $A_c(x) = \int_0^1 DF(sx)\,ds$, where the integration of the matrix $DF(x)$ is made term by term. In fact, we prove that LaSalle condition for $A_c(x)$ is a sufficient condition to get the global attraction of the origin when $m = 1$, or when $m = 2$ and $F$ is polynomial. We also show that this is no more true for $m = 2$ when $F$ is a rational map or when $m \geq 3$. Finally we consider the equivalent question for ordinary differential equations.

## 1. **Introduction and main results.**

LaSalle, in his book "The stability of dynamical systems"(see [13]), gives four conditions which imply conditions which imply that the origin of a discrete dynamical system defined on $\mathbb{R}$ is a global attractor, *i.e.* that the origin of $\mathbb{R}$ is the only fixed point and that all orbits go to it in forward time. He also proposes to study the natural extension of these conditions to $\mathbb{R}^m$, for discrete dynamical

$$x^{(n+1)} = F(x^{(n)}), \tag{1}$$

where $x^{(n)} \in \mathbb{R}^m$ and $F$ is a $\mathcal{C}^1$ map. Assuming that the only fix point is the origin, one of his suggestions is to write the map $F$, which defines the discrete dynamical system, as

$$F(x) = A(x)x, \tag{2}$$

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