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Simple Non-Autonomous Differential Equations With Many Limit Cycles

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Abstract

Consider the family of differential equations on the cylinder, $\frac{dx}{dt} = a(t) + b(t)|x|$, where $x, t \in \mathbb{R}$, and a, b are real, 1-periodic and smooth functions. The solutions satisfying $x(0) = x(1)$ are called periodic orbits of the equation. The periodic orbits that are isolated in the set of all the periodic orbits are usually called limit cycles. We give a proof, which is self contained, that there is no upper bound for the number of limit cycles of the above type of equations.

Key words: Non-smooth differential equations, bifurcations, limit cycles.
AMS Subject Classification: 34C23, 34C25, 37G15.

1 Introduction and main result

Let

$$\frac{dx}{dt} = S(t, x), \qquad t \in \mathcal{S}^1 = \mathbb{R}/[0, 1],\ x \in \mathbb{R}, \tag{1}$$