DISCRETE AND CONTINUOUS
DYNAMICAL SYSTEMS
Volume **24**, Number **3**, July **2009**

doi:10.3934/dcds.2009.24.699

pp. **699–729**

QUADRATIC PERTURBATIONS OF A CLASS OF QUADRATIC REVERSIBLE SYSTEMS WITH TWO CENTERS

Bartomeu Coll

Dep. de Matemàtiques i Informàtica. Univ. de Illes Balears
07122-Palma de Mallorca, Spain

Chengzhi Li

School of Mathematical Sciences and LMAM, Peking University
Beijing, 100871, China

Rafel Prohens

Dep. de Matemàtiques i Informàtica. Univ. de Illes Balears
07122-Palma de Mallorca, Spain

Dedicated to Professor Peter Bates on the occasion of his sixtieth birthday

Abstract. Quadratic perturbations of a one-parameter family of quadratic reversible systems with two centers (without other singularities in finite plane) are studied. The exact upper bound of the number of limit cycles, the configurations of limit cycles, and the bifurcation diagrams for different range of the parameter are given.

1. **Introduction.** The weak Hilbert 16th problem was solved completely in the quadratic case; that is, the least upper bound of the number of zeros of the Abelian integrals associated with quadratic perturbations of quadratic Hamiltonian systems is known. See [5, 9, 10, 14, 18] and the references therein.

The next natural step is to consider the same problem for quadratic integrable but non-Hamiltonian systems. Most mathematicians working in this field believe that perturbations of the quadratic reversible systems, especially with two centers, may give the richest dynamical behavior, see [1] for instance. We refer to [3, 17] for the detailed description of the weak Hilbert 16th problem, and to [11, 16, 19] for the classification of quadratic integrable systems.

In this paper we consider quadratic perturbations of a one-parameter family of quadratic reversible non-Hamiltonian systems, with two non-degenerate centers and without other singularities in the finite plane. For such family, we give an upper bound of the limit cycles that emerge from each of the two period annuli and we prove that it is sharp. Even more, we present the exact configuration of limit cycles that can be and the corresponding bifurcation diagrams.

2000 *Mathematics Subject Classification.* Primary: 34C07, 34C08; Secondary: 37G15.

Key words and phrases. Quadratic Reversible system, Abelian integral, limit cycle.

The first author is partially supported by grant DGICYT-MTM2005-08567. The second author is partially supported by grants NSFC-10671005 and NFSC-10831003. The third author is partially supported by grant UIB-2006.