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Limit Cycles for a Quadratic System with an Invariant Straight Line and Some Evolution of Phase Portraits

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In this work we study the uniqueness of limit cycles for quadratic systems with an invariant straight line. Then we investigate the phase portrait on the Poincaré-sphere.

1. Limit cycles

The goal of the first part is the following result:

Theorem 1. *If a quadratic system has an invariant straight line, then it has at most one limit cycle (stable or unstable).*

This theorem follows from the results of Cherkas (see [Ch]), Cherkas-Zhilevich (see [CZ1] and [CZ2]), and Rychkov (see [R]); see also Chen Lan Sun (see [Che]), and the book of Ye Yangian (see [Y]), where there is sketch of the proof. Coppel gave a shorter and simpler proof in [C3]. Previous to this work Coppel (see [C2]) suggested another proof also simpler and cleaver with some gaps. Here, we completes the proof of Coppel.

Working with quadratic systems with an invariant straight line we shall use the next result.

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This paper is in final form and no version of it will be submitted for publication elsewhere.