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Limit cycles bifurcating from a perturbed quartic center

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ABSTRACT

We consider the quartic center $\dot{x} = -yf(x,y), \dot{y} = xf(x,y)$, with $f(x,y) = (x+a)(y+b)(x+c)$ and $abc \neq 0$. Here we study the maximum number σ of limit cycles which can bifurcate from the periodic orbits of this quartic center when we perturb it inside the class of polynomial vector fields of degree n, using the averaging theory of first order. We prove that $4[(n-1)/2] + 4 \leqslant \sigma \leqslant 5[(n-1)/2] + 14$, where $[\eta]$ denotes the integer part function of η.

1. Introduction and statement of the main results

In the qualitative theory of real planar polynomial differential systems the problem of studying the number of limit cycles by perturbing the periodic orbits of a center has been extensively considered in literature. Basically, four methods have been used to perform such studies and they are based on: the Poincaré return map (see for instance [7,8,17]), the Poincaré–Pontrjagin–Melnikov integrals or Abelian integrals that are equivalent in the plane (see [2–4,6,9,10,23,24]), the inverse integrating factor (see [11–13,22]), and the averaging method which in the plane is also equivalent to the Abelian integrals (see for instance [5,16,18]).

Roughly speaking the averaging method gives a quantitative relation between the solutions of a non-autonomous periodic differential system and the solutions of its averaged differential system, which is autonomous. In particular the number of hyperbolic equilibrium points of the averaged differential system up to first order gives a lower bound of the maximum number of limit cycles of the non-autonomous periodic differential system, for more details see Theorem 2.6.1 of Sanders and Verhulst [20] and Theorem 11.5 of Verhulst [21]. Whenever the first averaged function vanishes, the number of limit cycles depends on the second averaged function, and so on (see for more details [5]). In some cases by using the second order averaging method the number of limit cycles increases, even more than the double see for instance [19].

Using the averaging theory of first order in [6], and the Melnikov method in [3], the authors give upper and lower bounds for the maximum number of limit cycles bifurcating from the period annulus of a cubic or quintic center, respectively. The lower bounds in both cases are $3[(n-1)/2] + 2$ and $3[(n+1)/2] + 1$, respectively.

In this paper we bound the maximum number of limit cycles that bifurcate from the period annulus surrounding the origin of the quartic polynomial differential system

$$\begin{aligned} \dot{x} &= -y(x+a)(y+b)(x+c), \\ \dot{y} &= x(x+a)(y+b)(x+c), \end{aligned} \tag{1}$$

where $a, b, c \in \mathbb{R} \setminus \{0\}$, when we perturb it inside the class of all polynomial differential systems of degree $n \in \mathbb{N}$ and using the averaging theory of first order. That is, we want to study the maximum number of limit cycles of the differential systems

$$\begin{aligned} \dot{x} &= -y(x+a)(y+b)(x+c) + \varepsilon P_n(x,y), \\ \dot{y} &= x(x+a)(y+b)(x+c) + \varepsilon Q_n(x,y), \end{aligned} \tag{2}$$

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