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On the Bifurcation of Limit Cycles Due to Polynomial Perturbations of Hamiltonian Centers

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Abstract. We study the number of limit cycles bifurcating from the period annulus of a real planar polynomial Hamiltonian ordinary differential system with a center at the origin when it is perturbed in the class of polynomial vector fields of a given degree.

Mathematics Subject Classification. Primary 34C05; Secondary 37C10.

Keywords. Ordinary differential system, polynomial system, planar system, Hamiltonian system, center, limit cycle, Melnikov function.

1. Introduction and Statement of the Main Results

In the qualitative theory of real planar polynomial differential systems, one of the main problems is the determination of limit cycles of a given vector field. The notion of limit cycle goes back to Poincaré, see [14–17]. He defined a limit cycle for a vector field in the plane as a periodic orbit of the differential system isolated in the set of all periodic orbits. The first works in determining the number of limit cycles of a given vector field can be traced back to Liénard [11] and Andronov [1]. After these works, the detection of the number of limit cycles of a polynomial differential system, intrinsically related to the so-called 16th Hilbert problem [7–9], has been extensively studied in the mathematical community, see, for instance, the books [3,19] and the papers [5,6,12,13].

One of the main tools of producing limit cycles is perturbing a system having a center. The notion of center goes back to Poincaré, see [14–17], who defined a center for a vector field on the real plane as a singular point having a neighborhood filled with periodic orbits with the exception of the singular point. If a system has a center, then when we perturb it, we may have a limit cycle that bifurcates in the perturbed system from some of the periodic orbits forming a center. This tool is one of the most effective ways of producing limit cycles, but it requires the knowledge of the first integral of the unperturbed system (the one having a center). It is well known that the determination of first integrals is also a very hard problem. This is why in this paper, we will

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