International Journal of Bifurcation and Chaos, Vol. 9, No. 9 (1999) 1751–1761

THE CENTER PROBLEM FOR DISCONTINUOUS LIÉNARD DIFFERENTIAL EQUATION

B. COLL* and R. PROHENS
Departament de Matemàtiques i Informàtica,
Facultat de Ciències, Universitat de les Illes Balears,
7071 Palma de Mallorca, Spain

A. GASULL†
Departament de Matemàtiques, Edifici Cc,
Universitat Autònoma de Barcelona,
08193 Bellaterra, Barcelona, Spain

Received October 10, 1997; Revised February 9, 1998

We prove that the Lyapunov constants for differential equations given by a vector field with a line of discontinuities are quasi-homogeneous polynomials. This property is strongly used to derive the general expression of the Lyapunov constants for two families of discontinuous Liénard differential equations, modulus some unknown coefficients. In one of the families these coefficients are determined and the center problem is solved. In the other family the center problem is just solved by assuming that the coefficients which appear in these expressions are nonzero. This assumption on the coefficients is supported by their computation (analytical and numerical) for several cases.

1. Introduction and Main Results

Planar differential equations given by a vector field which is either discontinuous or nonsmooth on a line often appear in applications (see e.g. [Andronov *et al.*, 1966] or [Popp *et al.*, 1995]). A general theory for such systems is given in [Filippov, 1988]. The main purpose of this paper is to give the center conditions and study the stability of the origin for two families of discontinuous Liénard differential equations.

Both families of differential equations can be written as a second-order differential equation of the form

$$\ddot{x} + f(x, \dot{x})\dot{x} + g(x) = 0\,,$$

with $g(x) = x$ and $f(x, \dot{x})$ being discontinuous on a line. The above equation was first studied by Levinson and Smith [1942] for f and g smooth functions. The case in which $f(x, \dot{x})$ is discontinuous on a line appears for some models of valve generators presented in [Andronov *et al.*, 1966, Chap. IX.7] or for some other models of damped vibrations in a media with hydraulic resistance given in [Sansone & Conti, 1964, pp. 341–345].

On the other hand, the center problem for special discontinuous families is studied in several papers (see e.g. [Lunkevich, 1968] or [Pleshkan & Sibirskii, 1973]) but as far as we know the case of Liénard equations is not yet solved.

As in the smooth case, the solution of the center problem and the knowledge of the stability of the origin for any parametric family will be reduced to the knowledge of some polynomials involving the parameters of the differential equation, the so-called *Lyapunov constants*.

*E-mail: dmitcv0@PS.uib.es
†E-mail: gasull@mat.uab.es