Generalized rational first integrals of analytic differential systems

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A R T I C L E   I N F O

A B S T R A C T

In this paper we mainly study the necessary conditions for the existence of functionally independent generalized rational first integrals of ordinary differential systems via the resonances. The main results extend some of the previous related ones, for instance the classical Poincaré's one (Poincaré, 1891, 1897 [16]), the Furta's one (Furta, 1996 [8]), part of Chen et al.'s ones (Chen et al., 2008 [4]), and the Shi's one (Shi, 2007 [18]). The key point in the proof of our main results is that functionally independence of generalized rational functions implies the functionally independence of their lowest order rational homogeneous terms.

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1. Introduction and statement of the main results

The rational first integrals in analytic differentiable systems and mainly in the particular case of polynomial differentiable systems have been studied intensively, specially inside the Darboux theory of integrability, see for instance [15,9,17,19]. In this paper we want to study the generalized rational first integrals of the analytic differential systems.

Consider analytic differential systems in $(\mathbb{C}^n, 0)$

$$\dot{x} = f(x), \quad x \in (\mathbb{C}^n, 0). \tag{1}$$

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