Universitat Autònoma de Barcelona

Facultat de Ciències

Departament de Matemàtiques

ON A NEW CLASS OF PERIODIC ORBITS FOR THE THREE-DIMENSIONAL ISOSCELES THREE–BODY PROBLEM

Montse Corbera Subirana

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del doctor Jaume Llibre
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1 Introduction

Using the method of analytic continuation we find sufficient conditions for the continuation of symmetric periodic orbits from the elliptic Sitnikov problem to the three–dimensional isosceles three–body problem (here, simply called *isosceles problem*), a particular case of the spatial three–body problem. The isosceles problem involves three masses, two equal and fixed (called *primaries*), and the third one denoted by μ. The analytic continuation of the symmetric periodic orbit is made by using the mass parameter μ. When $\mu = 0$ the isosceles problem becomes the elliptic Sitnikov problem, which goes to the circular Sitnikov problem when the eccentricity of the elliptic orbit described by the primaries tends to zero.

More explicitly we will find sufficient conditions to continue symmetric periodic orbits from the elliptic Sitnikov problem to the isosceles one that preserves either some initial conditions, the period or the total energy. Finally, using numerical computations we will find examples where these conditions are satisfied. That is, we will present symmetric periodic orbits of the elliptic Sitnikov problem that can be analytically continued.

The structure of the paper is the following one.

In Section 2 we define the problem, and the equations of motion in cylindrical coordinates with origin at the center of mass. In Section 3 we study the symmetry of the problem in this coordinate system, and we define symmetric periodic orbits. In Section 4 we summarise the basic results on the Kepler problem that we will need later on. In Section 5 we define some restricted problems associated to the isosceles problem that we will use for doing the computations. These restricted problems are the circular Sitnikov problem and the elliptic Sitnikov problem which are the only ones that have periodic orbits involving the three bodies.

In Section 6 we study the periodic orbits of the circular Sitnikov problem, the ones associated to the infinitessimal body, and the ones involving the three bodies. We also analyze their periods, energy and initial conditions. All these quantities will be used later on for the analytic continuation of the symmetric periodic orbits of the circular Sitnikov problem involving the three bodies to symmetric periodic orbits of the isosceles problem, see Section 7.

In Section 8 we find sufficient conditions in order to continue analytically symmetric periodic orbits from the elliptic Sitnikov problem to symmetric periodic orbits of the isosceles problem having the same initial velocity for the smallest mass. These sufficient conditions can be found in Theorem 8.3. This theorem only is valid when the symmetric periodic orbits of the elliptic Sitnikov problem are very close to symmetric periodic orbits of the circular Sitnikov problem involving the three bodies; that is, when the symmetric periodic orbit of the elliptic Sitnikov problem satisfies Lemma 8.2.

We find sufficient conditions in order to continue analytically symmetric periodic orbits from the elliptic Sitnikov problem to symmetric periodic orbits of the isosceles problem having the same inital position for the primaries (Section 9), the same period (Section 10) and the same total energy respectively (Section 11). These sufficient conditions can be found in Theorems 9.1, 10.1 and 11.1 respectively. We remark that in these cases the symmetric periodic orbit of the elliptic Sitnikov problem do not need to be close to a symmetric periodic orbit of the circular Sitnikov problem involving the three bodies.

In Appendix A we describe a program that, for a given symmetric periodic orbit of the elliptic Sitnikov problem, compute numerically the conditions of Theorem 8.3, then if these conditions are satisfied this obit can be analytically continued to a symmetric periodic orbit of the isosceles problem having the same initial velocity for the smallest mass.

In Appendix B we describe a program that compute numerically the conditions of Theorems 9.1, 10.1 and 11.1 for symmetric peridic orbits of the elliptic Sitnikov problem that are very close to periodic orbits of the circular Sitnikov problem involving the three bodies, that is, symmetric periodic orbits that satisfies Lemma 8.2. We see that for this kind of symmetric periodic orbits the conditions of Theorems 9.1, 10.1 and 11.1 are reduced to the same condition, then if this condition is satisfied for a symmetric periodic orbit of the ellitptic Sitnikov problem this orbit can be analytically continued to a family of symmetric periodic orbits of the isosceles problem having either the same initial position of the primaries, the same period, or the same total energy.