SIAM J. MATH. ANAL.
Vol. 47, No. 2, pp. 1377–1404


# BIFURCATION OF RELATIVE EQUILIBRIA OF THE (1+3)-BODY PROBLEM[*]

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**Abstract.** We study the relative equilibria of the limit case of the planar Newtonian 4-body problem when three masses tend to zero, the so-called (1+3)-body problem. Depending on the values of the infinitesimal masses the number of relative equilibria varies from ten to fourteen. Six of these relative equilibria are always convex, and the others are concave. Each convex relative equilibrium of the (1+3)-body problem can be continued to a unique family of relative equilibria of the general 4-body problem when three of the masses are sufficiently small and every convex relative equilibrium for these masses belongs to one of these six families.

**Key words.** celestial mechanics, relative equilibria, $(1+n)$-body problem

**AMS subject classifications.** 70F10, 70F15, 37N05, 70K42, 70K50

**DOI.** 10.1137/140978661

**1. Introduction.** A configuration of the $N$-body problem is *central* if the acceleration vector for each body is a common scalar multiple of its position vector (with respect to the center of mass). The planar central configurations are often called *relative equilibria*, that is, solutions of the $N$-body problem that remain fixed in a rotating frame.

The planar central configurations of the $N$-body problem are completely known only for $N = 2, 3$. Counting up to rotations and translations in the plane, there is a unique class of central configurations when $N = 2$, and there are exactly five classes of central configurations for each choice of three positive masses when $N = 3$: the two classes of equilateral triangle central configurations found in 1772 by Lagrange [22] and the three classes of collinear central configurations found in 1767 by Euler [16].

On the number of classes of central configurations of the $N$-body problem when $N > 3$ there are only partial results. Thus there are exactly $N!/2$ classes of collinear central configurations for a given set of $N$ positive masses; see Moulton [32]. Using Morse theory Palmore obtained a lower bound of the number of central configurations under a nondegeneracy assumption [33]. For $N = 4$, there are 12 collinear central configurations, and Palmore's lower bound is 34.

[*]Received by the editors July 21, 2014; accepted for publication (in revised form) January 27, 2015; published electronically April 2, 2015. The research of the first three authors was partially supported by MINECO/FEDER grants MTM2008-03437 and MTM2013-40998-P. The research of the second and third authors was partially supported by AGAUR grant 2014SGR 568.
http://www.siam.org/journals/sima/47-2/97866.html

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