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Trapezoid central configurations

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ABSTRACT

We classify all planar four–body central configurations where two pairs of the bodies are on parallel lines. Using cartesian coordinates, we show that the set of four–body trapezoid central configurations with positive masses forms a two–dimensional surface where two symmetric families, the rhombus and isosceles trapezoid, are on its boundary. We also prove that, for a given position of the bodies, in some cases a specific order of the masses determines the geometry of the configuration, namely acute or obtuse trapezoid central configuration. We also prove the existence of non–symmetric trapezoid central configurations with two pairs of equal masses.

1. Introduction

Central configurations are particular positions of the masses in the Newtonian n–body problem, where the position and acceleration vectors with respect to the center of masses are proportional, with the same constant of proportionality for all masses. They play an important role in celestial mechanics because, among other properties, they generate the unique known explicit solutions in the n–body problem for $n \geq 3$. For general information about central configurations see for instance Albouy and Chenciner [3], Hagihara [19], Moeckel [27], Saari [32,33], Schmidt [35], Smale [37,38] and Wintner [39].

More precisely we consider the planar n–body problem

$$m_k \ddot{\mathbf{q}}_k = - \sum_{j=1, j\neq k}^{n} G m_k m_j \frac{\mathbf{q}_k - \mathbf{q}_j}{|\mathbf{q}_k - \mathbf{q}_j|^3},$$

$k = 1, \ldots, n$, being $\mathbf{q}_k \in \mathbb{R}^2$ the position vector of the punctual mass m_k in an inertial coordinate system, and G is the gravitational constant that we can take equal to one by choosing conveniently the unit of time. The *configuration space* of the planar n–body problem is

$$\mathcal{E} = \{(\mathbf{q}_1, \ldots, \mathbf{q}_n) \in \mathbb{R}^{2n} \ : \ \mathbf{q}_k \neq \mathbf{q}_j, \ \text{for } k \neq j\}.$$

A configuration of the n bodies $(\mathbf{q}_1, \ldots, \mathbf{q}_n) \in \mathcal{E}$ is *central* if there is a positive constant λ such that

$$\ddot{\mathbf{q}}_k = -\lambda\,(\mathbf{q}_k - \mathbf{c}),$$

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