A Four-Body Convex Central Configuration with Perpendicular Diagonals Is Necessarily a Kite

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Abstract

We prove that any four-body convex central configuration with perpendicular diagonals must be a kite configuration. The result extends to general power-law potential functions, including the planar four-vortex problem.

Key Words: Central configuration, n-body problem, n-vortex problem

1 Introduction

Central configurations are an important class of solutions in n-body problems. They lead directly to homographic motions, where the initial shape of the configuration is preserved throughout the orbit, and play an important role in the study of the topology of the integral manifolds. In applied settings, central configurations have proved useful for designing low-cost low-energy space missions [8] and have been discerned in numerical simulations of the eyewall in hurricanes [5].

Locating a central configuration involves solving a challenging set of nonlinear algebraic equations. One approach to making the problem more tractable is to impose a geometric constraint on the shape of the configuration. In [4], Roberts and Cors investigated four-body co-circular central configurations, where the bodies are assumed to lie on a common circle. Other constraints employed involve symmetry, such as assuming the configuration has an axis of symmetry [2], or that it consists of nested regular n-gons [3].

Recently, Li, Deng and Zhang have shown that the diagonals of an isosceles trapezoid central configuration are not perpendicular [7]. We extend this result further by proving that the *only* four-body convex central configurations with perpendicular diagonals are the kite configurations (symmetric with respect to a diagonal). Here, "convex" means that no body is contained in the convex hull of the other three bodies. We prove this result for general power-law potential functions as well as for the planar four-vortex problem. It is hoped that the techniques described in this paper will prove fruitful for tackling similar open problems in celestial mechanics [1].

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