# Four-body co-circular central configurations 

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Received 26 May 2011, in final form 25 October 2011
Published 11 January 2012
Online at stacks.iop.org/Non/25/343
Recommended by A Chenciner


#### Abstract

We classify the set of central configurations lying on a common circle in the Newtonian four-body problem. Using mutual distances as coordinates, we show that the set of four-body co-circular central configurations with positive masses is a two-dimensional surface, a graph over two of the exterior side-lengths. Two symmetric families, the kite and isosceles trapezoid, are investigated extensively. We also prove that a co-circular central configuration requires a specific ordering of the masses and find explicit bounds on the mutual distances. In contrast to the general four-body case, we show that if any two masses of a four-body co-circular central configuration are equal, then the configuration has a line of symmetry.


Mathematics Subject Classification: 70F10, 70F15, 37N05
(Some figures may appear in colour only in the online journal)

## 1. Introduction

The study of central configurations (c.c.s) in the Newtonian $n$-body problem has become a vibrant sub-field of celestial mechanics. While much is known about specific cases, usually involving symmetry or assuming one or more bodies is infinitesimally small, less is known about the general structure of the set of c.c.s. For example, finiteness of c.c.s (up to symmetry) was only recently established by Hampton and Moeckel [12] in the four-body problem, and by Albouy and Kaloshin [2] (except for masses in a codimension two subvariety) in the five-body problem.

In this work, we strive to deepen our understanding of the general set of c.c.s by fully classifying the four-body co-circular central configurations (c.c.c.s), that is, those four-body c.c.s which also lie on a common circle (see figure 1). According to Hampton [10], this

