SIAM J. Applied Dynamical Systems
Vol. 2, No. 2, pp. 219–237

Coorbital Periodic Orbits in the Three Body Problem*

Josep M. Cors† and Glen R. Hall‡

Abstract. We consider the dynamics of coorbital motion of two small moons about a large planet which have nearly circular orbits with almost equal radii. These moons avoid collision because they switch orbits during each close encounter. We approach the problem as a perturbation of decoupled Kepler problems as in Poincaré's periodic orbits of the first kind. The perturbation is large but only in a small region in the phase space. We discuss the relationship required among the small quantities (radial separation, mass, and minimum angular separation). Persistence of the orbits is discussed.

Key words. coorbital motion, periodic orbits of the first kind, three body problem

AMS subject classifications. 70F15, 70F07

PII. S1111111102411304

1. Introduction.

In 1981, the Voyager 1 space craft explored and photographed the neighborhood of Saturn (see Aksnes [1]). These photos captured two remarkable small moons subsequently named Janus and Epimetheus. The moons are remarkable because they are of nearly equal mass, and their orbits have nearly equal radii. Because their orbits are so close, their motion is called "coorbital." As implied by Kepler's laws, the inner moon has a smaller period and hence catches up to the outer moon. The moons avoid collision because their mutual gravitation causes them to switch orbits when they approach each other. Figure 1.1 shows a schematic drawing of the paths of the coorbital moons. Figure 1.2 shows computed orbits of the two moons in rotating coordinates. The speed of rotation of the coordinates is that of the orbit of average radius. In this paper, we investigate the dynamics of the motion of a pair of small, comparable mass, coorbital moons as solutions of the Newtonian three body problem.

Several authors have studied the dynamics of coorbital satellites in general and Janus and Epimetheus in particular. One approach is to treat one of the moons as a test particle of zero mass and think of the dynamics as that of the restricted three body problem. Coorbital motion in this situation is very interesting and has been extensively studied both analytically and numerically (see, for example, Salo and Yoder [12] and Llibre and Ollé [4]).

Since the small moons are of comparable mass, it is more natural to treat them equally in the analysis. Two groups have approached the problem from this point of view. One approach is to do a "matching" of two separate dynamics problems—an "outer" problem, where the small moons do not interact, and an "inner" problem, where the interactions dominate the

*Received by the editors July 18, 2002; accepted for publication (in revised form) by J. Meiss January 20, 2003; published electronically May 23, 2003.
http://www.siam.org/journals/siads/2-2/41130.html
†Departament de Matemàtica Aplicada III, Universitat Politècnica de Catalunya, 08240, Manresa, Spain (cors@eupm.upc.es). The work of this author was partially supported by DGICYT grant PB96-1153 and MEC grant PR97-39332370.
‡Department of Mathematics and Statistics, Boston University, Boston, MA 02215 (rockford@math.bu.edu).